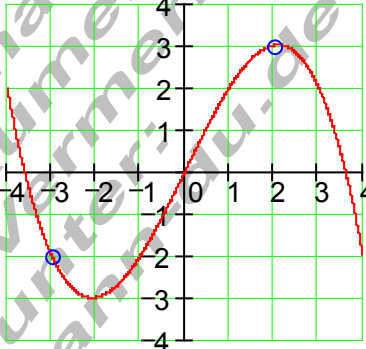


Lösungen Training ganzrationale Funktionen VIII

Ausführliche Lösungen:

A1	Aufgabe Finden Sie die Funktionsgleichung und skizzieren Sie den Graphen. grad 3, punktsymmetrisch $P_1(2 3)$ $P_2(-3 -2)$
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A1	Ausführliche Lösung grad 3, punktsymmetrisch durch $P_1(2 3)$ $P_2(-3 -2)$ Wegen der Punktsymmetrie hat die Funktionsgleichung nur ungerade Exponenten. Ansatz: $f(x) = a_3x^3 + a_1x$ $P_1(2 3) \Rightarrow f(2) = 8a_3 + 2a_1 = 3$ $P_2(-3 -2) \Rightarrow f(-3) = -27a_3 - 3a_1 = -2$ <table style="display: inline-table; vertical-align: middle;"> <tr> <td style="border-right: 1px solid black; padding: 2px;">a_1</td> <td style="padding: 2px;">a_3</td> <td style="border-right: 1px solid black; padding: 2px;"></td> <td style="padding: 2px;"></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">2</td> <td style="padding: 2px;">8</td> <td style="border-right: 1px solid black; padding: 2px;">3</td> <td style="padding: 2px;"> · 3</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">-3</td> <td style="padding: 2px;">-27</td> <td style="border-right: 1px solid black; padding: 2px;">-2</td> <td style="padding: 2px;"> · 2</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">6</td> <td style="padding: 2px;">24</td> <td style="border-right: 1px solid black; padding: 2px;">9</td> <td style="padding: 2px;"></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">-6</td> <td style="padding: 2px;">-54</td> <td style="border-right: 1px solid black; padding: 2px;">-4</td> <td style="padding: 2px;"> + </td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">6</td> <td style="padding: 2px;">24</td> <td style="border-right: 1px solid black; padding: 2px;">9</td> <td style="padding: 2px;"></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">0</td> <td style="padding: 2px;">-30</td> <td style="border-right: 1px solid black; padding: 2px;">5</td> <td style="padding: 2px;"></td> </tr> </table> $-30a_3 = 5$ $\Leftrightarrow a_3 = -\frac{1}{6}$ $6a_1 + 24a_3 = 9$ $\Leftrightarrow 6a_1 - 4 = 9$ $\Leftrightarrow a_1 = \frac{13}{6}$ $f(x) = -\frac{1}{6}x^3 + \frac{13}{6}x$	a_1	a_3			2	8	3	· 3	-3	-27	-2	· 2	6	24	9		-6	-54	-4	+	6	24	9		0	-30	5		$f(x) := -\frac{1}{6} \cdot x^3 + \frac{13}{6} \cdot x$  <p>Ganzrationale Funktion 3. Grades ist symmetrisch zum Ursprung, bzw, punktsymmetrisch mit drei Nullstellen.</p> <p>Bei Punktsymmetrie hat die Variable x nur ungerade Exponenten. Bei Achsensymmetrie sind sie gerade.</p>
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-3	-27	-2	· 2																											
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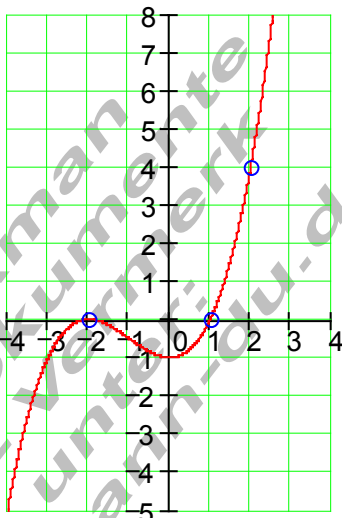
A2	Aufgabe
	Finden Sie die Funktionsgleichung und skizzieren Sie den Graphen. grad 3, Nullstellen $x_1 = -3$; $x_2 = -1$; $x_3 = 2$; $P(-2 2)$

A2	Ausführliche Lösung
grad 3, Nullstellen $x_1 = -3$; $x_2 = -1$; $x_3 = 2$ Punkt $P(-2 2)$ Ansatz über Linearfaktoren $f(x) = a_3(x - x_1)(x - x_2)(x - x_3)$ $f(x) = a_3(x + 3)(x + 1)(x - 2)$ Punktprobe: $f(-2) = 2 \Leftrightarrow a_3(-2 + 3)(-2 + 1)(-2 - 2) = 2$ $\Leftrightarrow 4a_3 = 2 \Leftrightarrow a_3 = \frac{1}{2}$ $f(x) = \frac{1}{2}(x + 3)(x + 1)(x - 2)$ $= \underline{\underline{\frac{1}{2}x^3 + x^2 - \frac{5}{2}x - 3}}$	$f(x) := \frac{1}{2} \cdot x^3 + x^2 - \frac{5}{2} \cdot x - 3$

A3	Aufgabe
	Finden Sie die Funktionsgleichung und skizzieren Sie den Graphen. grad 3, Nullstellen $x_{1/2} = 0$; $x_3 = 2$; $P(1 5)$

A3	Ausführliche Lösung	
	grad 3, Nullstellen $x_{1/2} = 0$; $x_3 = 2$; Punkt $P(1 5)$ Ansatz über Linearfaktoren $f(x) = a_3 x^2 (x - x_3)$ $f(x) = a_3 x^2 (x - 2)$ Punktprobe: $f(1) = 5 \Leftrightarrow a_3 \cdot 1^2 (1 - 2) = 5$ $\Leftrightarrow -a_3 = 5 \Leftrightarrow a_3 = -5$ $f(x) = -5x^2 (x - 2) = \underline{\underline{-5x^3 + 10x^2}}$	$f(x) := -5 \cdot x^3 + 10 \cdot x^2$

A4	Aufgabe
	Finden Sie die Funktionsgleichung und skizzieren Sie den Graphen. grad 3, Nullstellen $x_{1/2} = -2$; $x_3 = 1$; $P(2 4)$

A4	Ausführliche Lösung
grad 3, Nullstellen $x_{1/2} = -2$; $x_3 = 1$ Punkt $P(2 4)$	$f(x) := \frac{1}{4} \cdot x^3 + \frac{3}{4} \cdot x^2 - 1$
Ansatz über Linearfaktoren $f(x) = a_3 (x - x_{1/2})^2 (x - x_3)$ $f(x) = a_3 (x + 2)^2 (x - 1)$	
Punktprobe: $f(2) = 4 \Leftrightarrow a_3 \cdot (2+2)^2 (2-1) = 4$ $\Leftrightarrow 16a_3 = 4 \Leftrightarrow a_3 = \frac{1}{4}$	
$f(x) = \frac{1}{4} (x+2)^2 (x-1)$ $= \frac{1}{4} x^3 + \frac{3}{4} x^2 - 1$	

A5	Aufgabe Finden Sie die Funktionsgleichung und skizzieren Sie den Graphen. grad 3, Nullstellen $x_{1/2/3} = 3$; $P(-1 8)$
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A5	Ausführliche Lösung grad 3, Nullstellen $x_{1/2/3} = 3$ Punkt $P(-1 8)$ Ansatz über Linearfaktoren $f(x) = a_3(x - x_{1/2/3})^3$ $f(x) = a_3(x - 3)^2$ Punktprobe: $f(-1) = 8 \Leftrightarrow a_3 \cdot (-1 - 3)^3 = 8$ $\Leftrightarrow -64a_3 = 8 \Leftrightarrow a_3 = -\frac{1}{8}$ $f(x) = -\frac{1}{8}(x - 3)^3$ $= -\frac{1}{8}x^3 + \frac{9}{8}x^2 - \frac{27}{8}x + \frac{27}{8}$	$f(x) := \frac{-1}{8} \cdot x^3 + \frac{9}{8} \cdot x^2 - \frac{27}{8} \cdot x + \frac{27}{8}$
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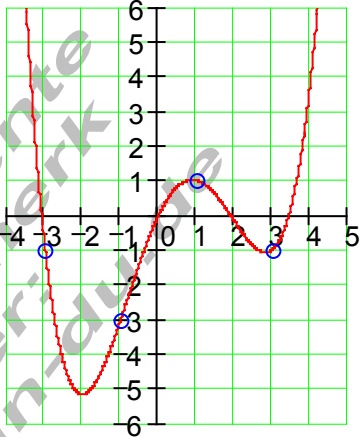
A6	Aufgabe
	Finden Sie die Funktionsgleichung und skizzieren Sie den Graphen. grad 4, achsensymmetrisch $P_1(1 2)$; $P_2(2 -1)$; $P_3(-3 -2)$

A6	Ausführliche Lösung																																																																	
	<p>grad 4, achsensymmetrisch $P_1(1 2)$; $P_2(2 -1)$; $P_3(-3 -2)$ Wegen der Achsensymmetrie hat die Funktionsgleichung nur gerade Exponenten. Ansatz: $f(x) = a_4x^4 + a_2x^2 + a_0$ $P_1(1 2) \Rightarrow f(1) = 1a_4 + 1a_2 + 1a_0 = 2$ $P_2(2 -1) \Rightarrow f(2) = 16a_4 + 4a_2 + 1a_0 = -1$ $P_3(-3 -2) \Rightarrow f(-3) = 81a_4 + 9a_2 + 1a_0 = -2$</p> <table border="1"> <thead> <tr> <th>a_0</th> <th>a_2</th> <th>a_4</th> <th></th> <th></th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1</td> <td>1</td> <td>2</td> <td></td> </tr> <tr> <td>1</td> <td>4</td> <td>16</td> <td>-1</td> <td>II-I</td> </tr> <tr> <td>1</td> <td>9</td> <td>81</td> <td>-2</td> <td>III-I</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> <td>2</td> <td></td> </tr> <tr> <td>0</td> <td>3</td> <td>15</td> <td>-3</td> <td> :3</td> </tr> <tr> <td>0</td> <td>8</td> <td>80</td> <td>-4</td> <td> :8</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> <td>2</td> <td></td> </tr> <tr> <td>0</td> <td>1</td> <td>5</td> <td>-1</td> <td></td> </tr> <tr> <td>0</td> <td>1</td> <td>10</td> <td>-1/2</td> <td>III-II</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> <td>2</td> <td></td> </tr> <tr> <td>0</td> <td>1</td> <td>5</td> <td>-1</td> <td></td> </tr> <tr> <td>0</td> <td>0</td> <td>5</td> <td>1/2</td> <td></td> </tr> </tbody> </table>	a_0	a_2	a_4			1	1	1	2		1	4	16	-1	II-I	1	9	81	-2	III-I	1	1	1	2		0	3	15	-3	:3	0	8	80	-4	:8	1	1	1	2		0	1	5	-1		0	1	10	-1/2	III-II	1	1	1	2		0	1	5	-1		0	0	5	1/2	
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		$5a_4 = \frac{1}{2} \quad :5 \Leftrightarrow a_4 = \frac{1}{10}$ $a_2 + 5a_4 = -1$ $\Leftrightarrow a_2 + \frac{5}{10} = -\frac{10}{10} \quad -\frac{5}{10}$ $\Leftrightarrow a_2 = -\frac{15}{10} = -\frac{3}{2}$ $a_0 + a_2 + a_4 = 2$ $\Leftrightarrow a_0 - \frac{3}{2} + \frac{1}{10} = \frac{20}{10} \quad +\frac{15}{10} - \frac{1}{10}$ $\Leftrightarrow a_0 = \frac{34}{10} = \frac{17}{5}$ $f(x) = \frac{1}{10}x^4 - \frac{3}{2}x^2 + \frac{17}{5}$																																																																

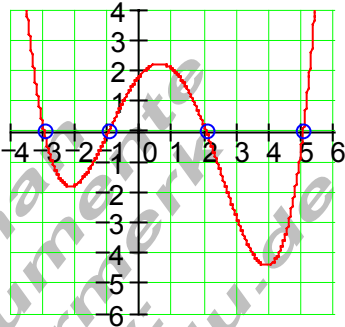
A7	Aufgabe
	Finden Sie die Funktionsgleichung und skizzieren Sie den Graphen. grad 4, Nullstellen $x_{1/2/3} = -2$; $x_4 = 2$; $P(1 3)$

A7	Ausführliche Lösung
	<p>grad 4, Nullstellen $x_{1/2/3} = -2$; $x_4 = 2$ Punkt $P(1 3)$</p> <p>Ansatz über Linearfaktoren</p> $f(x) = a_4 (x - x_{1/2/3})^3 (x - x_4)$ $f(x) = a_4 (x + 2)^3 (x - 2)$ <p>Punktprobe :</p> $f(1) = 3 \Leftrightarrow a_4 (1 + 2)^3 (1 - 2) = 3$ $\Leftrightarrow -27a_4 = 3 \Leftrightarrow a_4 = -\frac{1}{9}$ $f(x) = -\frac{1}{9} (x + 2)^3 (x - 2)$ $= \underline{\underline{-\frac{1}{9}x^4 - \frac{4}{9}x^3 + \frac{16}{9}x + \frac{16}{9}}}$
	$f(x) := \frac{-1}{9} \cdot x^4 - \frac{4}{9} \cdot x^3 + \frac{16}{9} \cdot x + \frac{16}{9}$

A8	Aufgabe
	Finden Sie die Funktionsgleichung und skizzieren Sie den Graphen. grad 4, durch den Ursprung $P_1(1 1)$; $P_2(-1 -3)$; $P_3(3 -1)$; $P_4(-3 -1)$

A8	Ausführliche Lösung																																																																																					
	<p>grad 4, durch den Ursprung ($a_0 = 0$) $P_1(1 1)$; $P_2(-1 -3)$; $P_3(3 -1)$; $P_4(-3 -1)$ Ansatz: $f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x$ $P_1(1 1) : f(1) = 1a_4 + 1a_3 + 1a_2 + 1a_1 = 1$ $P_2(-1 -3) : f(-1) = 1a_4 - 1a_3 + 1a_2 - 1a_1 = 3$ $P_3(3 -1) : f(3) = 81a_4 + 27a_3 + 9a_2 + 3a_1 = -1$ $P_4(-3 -1) : f(-3) = 81a_4 - 27a_3 + 9a_2 - 3a_1 = -1$</p> <table style="border-collapse: collapse; margin-left: 20px;"> <thead> <tr> <th style="border-right: 1px solid black; padding: 2px;">a_1</th> <th style="border-right: 1px solid black; padding: 2px;">a_2</th> <th style="border-right: 1px solid black; padding: 2px;">a_3</th> <th style="border-right: 1px solid black; padding: 2px;">a_4</th> <th style="padding: 2px;"></th> </tr> </thead> <tbody> <tr> <td style="border-right: 1px solid black; padding: 2px;">1</td> <td style="border-right: 1px solid black; padding: 2px;">1</td> <td style="border-right: 1px solid black; padding: 2px;">1</td> <td style="border-right: 1px solid black; padding: 2px;">1</td> <td style="padding: 2px;">1</td> </tr> <tr> <td style="border-right: 1px solid black; 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padding: 2px;">84</td> <td style="padding: 2px;">2 IV-6·II</td> </tr> <tr style="border-top: 1px solid black;"> <td style="border-right: 1px solid black; padding: 2px;">1</td> <td style="border-right: 1px solid black; padding: 2px;">1</td> <td style="border-right: 1px solid black; padding: 2px;">1</td> <td style="border-right: 1px solid black; padding: 2px;">1</td> <td style="padding: 2px;">1</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">0</td> <td style="border-right: 1px solid black; padding: 2px;">2</td> <td style="border-right: 1px solid black; padding: 2px;">0</td> <td style="border-right: 1px solid black; padding: 2px;">2</td> <td style="padding: 2px;">-2</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">0</td> <td style="border-right: 1px solid black; padding: 2px;">0</td> <td style="border-right: 1px solid black; padding: 2px;">24</td> <td style="border-right: 1px solid black; padding: 2px;">72</td> <td style="padding: 2px;">2</td> </tr> <tr> <td style="border-right: 1px solid black; 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	$f(x) := \frac{1}{9} \cdot x^4 - \frac{1}{4} \cdot x^3 - \frac{10}{9} \cdot x^2 + \frac{9}{4} \cdot x$ 																																																																																					
	$144a_4 = 16 \Leftrightarrow a_4 = \frac{1}{9}$ $24a_3 + 72a_4 = 2$ $\Leftrightarrow 24a_3 + \frac{72}{9} = \frac{18}{9} \Leftrightarrow a_3 = -\frac{1}{4}$ $2a_2 + 2a_4 = -2$ $\Leftrightarrow 2a_2 + \frac{2}{9} = -\frac{18}{9} \Leftrightarrow a_2 = -\frac{10}{9}$ $a_1 + a_2 + a_3 + a_4 = 1$ $\Leftrightarrow a_1 - \frac{10}{9} - \frac{1}{4} + \frac{1}{9} = 1 \Leftrightarrow a_1 = \frac{9}{4}$ $f(x) = \frac{1}{9}x^4 - \frac{1}{4}x^3 - \frac{10}{9}x^2 + \frac{9}{4}x$																																																																																					

A9	Aufgabe
	Finden Sie die Funktionsgleichung und skizzieren Sie den Graphen. grad 4, Nullstellen $x_1 = -3$; $x_2 = -1$; $x_3 = 2$; $x_4 = 5$; $P(1 2)$

A9	Ausführliche Lösung
<p>grad 4, Nullstellen $x_1 = -3$; $x_2 = -1$; $x_3 = 2$; $x_4 = 5$ Punkt $P(1 2)$ Ansatz über Linearfaktoren $f(x) = a_4(x - x_1)(x - x_2)(x - x_3)(x - x_4)$ $f(x) = a_4(x + 3)(x + 1)(x - 2)(x - 5)$ Punktprobe : $f(1) = 2$ $\Leftrightarrow a_4(1+3)(1+1)(1-2)(1-5) = 2$ $\Leftrightarrow 32a_4 = 2 \Leftrightarrow a_4 = \frac{1}{16}$ $f(x) = \frac{1}{16}(x+3)(x+1)(x-2)(x-5)$ $= \frac{1}{16}x^4 - \frac{3}{16}x^3 - \frac{15}{16}x^2 + \frac{19}{16}x + \frac{15}{8}$</p>	$f(x) = \frac{1}{16}x^4 - \frac{3}{16}x^3 - \frac{15}{16}x^2 + \frac{19}{16}x + \frac{15}{8}$ 

A10	Aufgabe
	Finden Sie die Funktionsgleichung und skizzieren Sie den Graphen. grad 4, Nullstellen $x_{1/2} = -3$; $x_{3/4} = 2$; $P(1 4)$

A10	Ausführliche Lösung
<p>grad 4, Nullstellen $x_{1/2} = -3$; $x_{3/4} = 2$; $P(1 4)$ Ansatz über Linearfaktoren $f(x) = a_4(x - x_{1/2})^2(x - x_{3/4})^2$ $f(x) = a_4(x + 3)^2(x - 2)^2$ Punktprobe : $f(1) = 4 \Leftrightarrow a_4(1+3)^2(1+2)^2 = 4$ $\Leftrightarrow 16a_4 = 4 \Leftrightarrow a_4 = \frac{1}{4}$ $f(x) = \frac{1}{4}(x+3)^2(x-2)^2$ $= \frac{1}{4}x^4 + \frac{1}{2}x^3 - \frac{11}{4}x^2 - 3x + 9$</p>	$f(x) := \frac{1}{4} \cdot x^4 + \frac{1}{2} \cdot x^3 - \frac{11}{4} \cdot x^2 - 3 \cdot x + 9$ 