

Lösungen Potenzen VI (Potenzbrüche vereinfachen)**Ergebnisse:**

E1	Ergebnisse		
a)	$\frac{x^4 - x^3}{x^2 - x} = x^2$	b)	$\frac{(3x^2 - 6x^3)^2}{9x^4} = (1 - 2x)^2$
c)	$\frac{5x + 15}{x + 3} = 5$	d)	$\frac{a^3 + 2a^2b + ab^2}{(a + b)^2} = a$
e)	$\frac{a^4 - a^2b^2}{ab - a^2} = -a(a + b)$	f)	$\frac{e^{3x} + e^{2x}}{e^{2x}} = e^x + 1$

E2	Ergebnisse		
a)	$\frac{x^4 - 6x^3}{5x - 30} = \frac{1}{5}x^3$	b)	$\frac{k^3 + 6k^2 + 9k}{k^2 - 9} = \frac{k(k + 3)}{k - 3}$
c)	$\frac{a^7b^3 - ab^7}{a^5b - a^2b^4} = \frac{a^6b^2 - b^6}{a^4 - ab^3}$	d)	$\frac{x^{2n} - 10x^n + 25}{x^{2n} - 25} = \frac{x^n - 5}{x^n + 5}$
e)	$\frac{x^6 - k^2}{x^4 + kx} = \frac{x^3 - k}{x}$	f)	$\frac{x^{n+3} - x^{n+1}}{x^{n+1} + x^n} = x(x - 1)$

E3	Ergebnisse	
a)	$\frac{(x^2 + 8xy + 16y^2)}{(2x - 3y)^{-2}} \cdot \frac{x^2 - 16y^2}{2x - 3y} = \frac{x + 4y}{x - 4y} (2x - 3y)^3$	
b)	$\frac{4k^2 - 4}{k^2 + 2k + 1} = \frac{4(k - 1)}{k + 1}$	
c)	$\frac{x^{n-1} - x^n}{x^n - x^{n+2}} = \frac{1}{x(x + 1)}$	
d)	$\frac{3 + 6x^2}{2x} - \frac{6x^3 - 5}{3x^2} - \frac{2x^4 - 2}{2x^3} = \frac{9x^2 + 10x + 6}{6x^3}$	
e)	$\frac{32}{2^{n+5}} + \frac{2^{-n+3}}{8} = 2^{1-n}$	
f)	$\frac{2(a^2 + b^2)^2}{a^5 - ab^4} = \frac{2(a^2 + b^2)}{a(a^2 - b^2)}$	

E4	Ergebnisse
a)	$\frac{x^4 - x^3}{x^4 - x^2} = \frac{x}{x+1}$
b)	$\frac{x^3y - xy^5}{x^3y^2 - x^2y^4} = \frac{x+y^2}{xy}$
c)	$\frac{am - an + bm - bn}{a^2 - b^2} = \frac{m-n}{a-b}$

E5	Ergebnisse
a)	$y = \frac{1}{4}x^4 - 2kx^3 + \frac{9}{2}k^2x^2$ für $x = 3k \Rightarrow y = \frac{27k^4}{4}$
b)	$y = \frac{kx^3}{2(x+k)^2}$ für $x = -3k \Rightarrow y = -\frac{27k^2}{8}$
c)	$y = \frac{3}{2k^2}x^4 - \frac{4}{k}x^3 + 3x^2 - 4$ für $x = \frac{1}{3}k \Rightarrow y = \frac{11k^2}{54} - 4$
d)	$y = \frac{e^{3kx} + 4e^3}{kx - 4}$ für $x = -\frac{1}{k} \Rightarrow y = -\frac{e^{-3} + 4e^3}{5}$
e)	$y = e^{x^2 - k^2} + 3e^{5k - (k-x)}$ für $x = -k \Rightarrow y = 1 + 3e^{3k}$
f)	$y = \frac{x^3 - kx + 1}{x^3}$ für $x = \frac{3}{2k} \Rightarrow 1 - \frac{4k^3}{27}$

E6	Ergebnisse
a)	$a^n + a^{4-n} + a^{2n} = a^{2n} (a^{-n} + a^{4-3n} + 1)$
b)	$a^{k-2} + a^{3k} + a^{-k-1} = a^{-k} (a^{2k-2} + a^{4k} + a^{-1})$
c)	$a^3 + a^{1-n} + a^{n+4} = a^{n+3} (a^{-n} + a^{-2n-2} + a)$
d)	$\frac{3}{2}x^4 + \frac{3}{4}x^3 + \frac{1}{8}x^2 = \frac{1}{8}x^2 (12x^2 + 6x + 1)$
e)	$e^{3x} - 2e^{-x} = e^{-x} (e^{4x} - 2)$
f)	$ke^{2x} - 2e^{x+1} = e^x (ke^x - 2e)$

Potenzgesetze

$a^m \cdot a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$	$a^n \cdot b^n = (a \cdot b)^n$	$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$
$(a^n)^m = a^{n \cdot m}$	$\sqrt[n]{a^m} = a^{\frac{m}{n}}$	$a^0 = 1$	$\frac{1}{a^n} = a^{-n}$

Ausführliche Lösungen :

A1	Aufgabe					
	Vereinfachen Sie					
	a)	$\frac{x^4 - x^3}{x^2 - x}$	b)	$\frac{(3x^2 - 6x^3)^2}{9x^4}$	c)	$\frac{5x + 15}{x + 3}$
d)	$\frac{a^3 + 2a^2b + ab^2}{(a + b)^2}$	e)	$\frac{a^4 - a^2b^2}{ab - a^2}$	f)	$\frac{e^{3x} + e^{2x}}{e^{2x}}$	

A1	Ausführliche Lösungen						
	a)	$\frac{x^4 - x^3}{x^2 - x} = \frac{x^3(x-1)}{x(x-1)}$ $= \frac{x^3 \cancel{(x-1)}}{x \cancel{(x-1)}}$ $= \frac{x^3}{x} = \underline{\underline{x^2}}$	b)	$\frac{(3x^2 - 6x^3)^2}{9x^4} = \frac{[3x^2(1-2x)]^2}{9x^4}$ $= \frac{9x^4(1-2x)^2}{9x^4}$ $= \frac{\cancel{9x^4}(1-2x)^2}{\cancel{9x^4}} = \underline{\underline{(1-2x)^2}}$			

A1	Ausführliche Lösungen						
	c)	$\frac{5x + 15}{x + 3} = \frac{5(x+3)}{(x+3)}$ $= \frac{5 \cancel{(x+3)}}{\cancel{(x+3)}}$ $= \underline{\underline{5}}$	d)	$\frac{a^3 + 2a^2b + ab^2}{(a+b)^2} = \frac{a(a^2 + 2ab + b^2)}{(a+b)^2}$ $= \frac{a \cancel{(a+b)^2}}{\cancel{(a+b)^2}}$ $= \frac{a(a+b)^2}{(a+b)^2} = \underline{\underline{a}}$			

A1	Ausführliche Lösungen						
	e)	$\frac{a^4 - a^2b^2}{ab - a^2} = \frac{a^2(a^2 - b^2)}{a(b-a)}$ $= \frac{a(a-b)(a+b)}{-1(a-b)}$ $= \frac{a \cancel{(a-b)}(a+b)}{-1 \cancel{(a-b)}}$ $= \underline{\underline{-a(a+b)}}$	f)	$\frac{e^{3x} + e^{2x}}{e^{2x}} = \frac{e^{3x}}{e^{2x}} + \frac{e^{2x}}{e^{2x}}$ $= e^{3x-2x} + e^{2x-2x}$ $= e^x + \underbrace{e^0}_1$ $= \underline{\underline{e^x + 1}}$			

A2	Aufgabe					
	Vereinfachen Sie					
	a)	$\frac{x^4 - 6x^3}{5x - 30}$	b)	$\frac{k^3 + 6k^2 + 9k}{k^2 - 9}$	c)	$\frac{a^7b^3 - ab^7}{a^5b - a^2b^4}$
d)	$\frac{x^{2n} - 10x^n + 25}{x^{2n} - 25}$	e)	$\frac{x^6 - k^2}{x^4 + kx}$	f)	$\frac{x^{n+3} - x^{n+1}}{x^{n+1} + x^n}$	

A2	Ausführliche Lösungen					
	a)	$\frac{x^4 - 6x^3}{5x - 30} = \frac{x^3(x-6)}{5(x-6)}$ $= \frac{x^3 \cancel{(x-6)}}{5 \cancel{(x-6)}}$ $= \frac{1}{5}x^3$	b)	$\frac{k^3 + 6k^2 + 9k}{k^2 - 9} = \frac{k(k^2 + 6k + 9)}{(k-3)(k+3)}$ $= \frac{k(k+3)^2}{(k-3)(k+3)}$ $= \frac{k(k+3)}{k-3}$		

A2	Ausführliche Lösungen					
	c)	$\frac{a^7b^3 - ab^7}{a^5b - a^2b^4} = \frac{ab(a^6b^2 - b^6)}{ab(a^4 - ab^3)}$ $= \frac{a^6b^2 - b^6}{a^4 - ab^3}$	d)	$\frac{x^{2n} - 10x^n + 25}{x^{2n} - 25} = \frac{(x^n - 5)^2}{(x^n - 5)(x^n + 5)}$ $= \frac{x^n - 5}{x^n + 5}$		

A2	Ausführliche Lösungen					
	e)	$\frac{x^6 - k^2}{x^4 + kx} = \frac{(x^3 - k)(x^3 + k)}{x(x^3 + k)}$ $= \frac{x^3 - k}{x}$	f)	$\frac{x^{n+3} - x^{n+1}}{x^{n+1} + x^n} = \frac{x^n \cdot x(x^2 - 1)}{x^n(x+1)}$ $= \frac{x(x-1)(x+1)}{(x+1)}$ $= \underline{\underline{x(x-1)}}$		

A3	Aufgabe					
	Vereinfachen Sie					
	a)	$\frac{(x^2 + 8xy + 16y^2)}{(2x - 3y)^{-2}} : \frac{x^2 - 16y^2}{2x - 3y}$	b)	$\frac{4k^2 - 4}{k^2 + 2k + 1}$	c)	$\frac{x^{n-1} - x^n}{x^n - x^{n+2}}$
d)	$\frac{3 + 6x^2}{2x} - \frac{6x^3 - 5}{3x^2} - \frac{2x^4 - 2}{2x^3}$	e)	$\frac{32}{2^{n+5}} + \frac{2^{-n+3}}{8}$	f)	$\frac{2(a^2 + b^2)^2}{a^5 - ab^4}$	

A3 Ausführliche Lösungen	
a)	$\frac{(x^2 + 8xy + 16y^2)}{(2x - 3y)^{-2}} : \frac{x^2 - 16y^2}{2x - 3y}$ $= \frac{(x^2 + 8xy + 16y^2)(2x - 3y)}{(2x - 3y)^{-2}(x^2 - 16y^2)}$ $= \frac{(x + 4y)^2 (2x - 3y)^3}{(x - 4)(x + 4)}$ $= \frac{x + 4y}{x - 4y} (2x - 3y)^3$
b)	$\frac{4k^2 - 4}{k^2 + 2k + 1} = \frac{4(k^2 - 1)}{(k + 1)^2}$ $= \frac{4(k - 1)(k + 1)}{(k + 1)(k + 1)}$ $= \frac{4(k - 1) \cancel{(k + 1)}}{(k + 1) \cancel{(k + 1)}}$ $= \frac{4(k - 1)}{k + 1}$

A3 Ausführliche Lösungen	
c)	$\frac{x^{n-1} - x^n}{x^n - x^{n+2}} = \frac{x^{n-1}(1-x)}{x^n(1-x^2)}$ $= \frac{1}{x} \cdot \frac{(1-x)}{(1-x)(1+x)}$ $= \frac{1}{x} \cdot \frac{\cancel{(1-x)}}{\cancel{(1-x)}(1+x)}$ $= \frac{1}{x(x+1)}$
d)	$\frac{3 + 6x^2}{2x} - \frac{6x^3 - 5}{3x^2} - \frac{2x^4 - 2}{2x^3} \quad \text{HN} = 6x^3$ $\Rightarrow \frac{3x^2(3 + 6x^2)}{6x^3} - \frac{2x(6x^3 - 5)}{6x^3} - \frac{3(2x^4 - 2)}{6x^3}$ $= \frac{9x^2 + 18x^4 - 12x^4 + 10x - 6x^4 + 6}{6x^3}$ $= \frac{9x^2 + 10x + 6}{6x^3}$

A3 Ausführliche Lösungen	
e)	$\frac{32}{2^{n+5}} + \frac{2^{-n+3}}{8} = \frac{2^5}{2^{n+5}} + \frac{2^{-n+3}}{2^3}$ $= 2^{5-(n+5)} + 2^{-n+3-3}$ $= 2^{5-n-5} + 2^{-n}$ $= 2^{-n} + 2^{-n}$ $= 2 \cdot 2^{-n} = \underline{\underline{2^{1-n}}}$
f)	$\frac{2(a^2 + b^2)^2}{a^5 - ab^4} = \frac{2(a^2 + b^2)^2}{a(a^4 - b^4)}$ $= \frac{2(a^2 + b^2)^2}{a(a^2 - b^2)(a^2 + b^2)}$ $= \frac{2(a^2 + b^2)}{a(a^2 - b^2)}$

A4 Aufgabe					
Vereinfachen Sie					
a)	$\frac{x^4 - x^3}{x^4 - x^2}$	b)	$\frac{x^3y - xy^5}{x^3y^2 - x^2y^4}$	c)	$\frac{am - an + bm - bn}{a^2 - b^2}$

A4	Ausführliche Lösungen
a)	$\frac{x^4 - x^3}{x^4 - x^2} = \frac{x^3(x-1)}{x^2(x^2-1)} = \frac{x(x-1)}{(x-1)(x+1)} = \frac{x \cancel{(x-1)}}{\cancel{(x-1)}(x+1)} = \underline{\underline{\frac{x}{x+1}}}$

A4	Ausführliche Lösungen
b)	$\frac{x^3y - xy^5}{x^3y^2 - x^2y^4} = \frac{xy(x^2 - y^4)}{x^2y^2} = \frac{1}{xy} \cdot \frac{(x-y^2)(x+y^2)}{(x-y^2)} = \frac{1}{xy} \cdot \frac{\cancel{(x-y^2)}(x+y^2)}{\cancel{(x-y^2)}} = \underline{\underline{\frac{x+y^2}{xy}}}$

A4	Ausführliche Lösungen
c)	$\frac{am - an + bm - bn}{a^2 - b^2} = \frac{a(m-n) + b(m-n)}{(a-b)(a+b)} = \frac{(m-n)(a+b)}{(a-b)(a+b)} = \underline{\underline{\frac{m-n}{a-b}}}$

A5	Aufgabe
	Berechnen Sie y
a)	$y = \frac{1}{4}x^4 - 2kx^3 + \frac{9}{2}k^2x^2$ für $x = 3k$
b)	$y = \frac{kx^3}{2(x+k)^2}$ für $x = -3k$
c)	$y = \frac{3}{2k^2}x^4 - \frac{4}{k}x^3 + 3x^2 - 4$ für $x = \frac{1}{3}k$
d)	$y = \frac{e^{3kx} + 4e^3}{kx - 4}$ für $x = -\frac{1}{k}$
e)	$y = e^{x^2 - k^2} + 3e^{5k - (k-x)}$ für $x = -k$
f)	$y = \frac{x^3 - kx + 1}{x^3}$ für $x = \frac{3}{2k}$

A5	Ausführliche Lösungen
a)	$y = \frac{1}{4}x^4 - 2kx^3 + \frac{9}{2}k^2x^2$ <p>für $x = 3k$ gilt:</p> $y = \frac{1}{4}(3k)^4 - 2k(3k)^3 + \frac{9}{2}k^2(3k)^2$ $= \frac{1}{4} \cdot 3^4 \cdot k^4 - 2 \cdot 3^3 \cdot k^4 + \frac{3^2 \cdot 3^2}{2} k$ $= \left(\frac{1}{4} \cdot 3^4 - 2 \cdot 3^3 + \frac{1}{2} \cdot 3^4 \right) k^4$ $= 3^3 \left(\frac{1}{4} \cdot 3 - 2 + \frac{1}{2} \cdot 3 \right) k^4$ $= 27 \left(\frac{3}{4} - \frac{8}{4} + \frac{6}{4} \right) k^4$ $= 27 \cdot \left(\frac{1}{4} \right) k^4 = \underline{\underline{\frac{27k^4}{4}}}$
b)	$y = \frac{kx^3}{2(x+k)^2}$ <p>für $x = -3k$ gilt:</p> $y = \frac{k(-3k)^3}{2(-3k+k)^2}$ $= \frac{-27k^4}{2 \cdot (-2k)^2}$ $= \frac{-27k^4}{2 \cdot 4k^2}$ $= \underline{\underline{-\frac{27}{8}k^2}}$

A5 Ausführliche Lösungen	
<p>c)</p> $y = \frac{3}{2k^2}x^4 - \frac{4}{k}x^3 + 3x^2 - 4$ <p>für $x = \frac{1}{3}k = \frac{k}{3}$ gilt:</p> $y = \frac{3}{2k^2} \left(\frac{k}{3}\right)^4 - \frac{4}{k} \left(\frac{k}{3}\right)^3 + 3 \left(\frac{k}{3}\right)^2 - 4$ $= \frac{3 \cdot k^4}{2 \cdot 3^4 k^2} - \frac{4k^3}{k \cdot 3^3} + 3 \cdot \frac{k^2}{3^2} - 4$ $= \frac{1}{54}k^2 - \frac{4}{27}k^2 + \frac{1}{3}k^2 - 4$ $= \frac{1}{54}k - \frac{8}{54}k^2 + \frac{18}{54}k^2 - 4$ $= \frac{11k^2}{54} - 4$	<p>d)</p> $y = \frac{e^{3kx} + 4e^3}{kx - 4}$ <p>für $x = -\frac{1}{k}$ gilt:</p> $y = \frac{e^{3k\left(-\frac{1}{k}\right)} + 4e^3}{k\left(-\frac{1}{k}\right) - 4}$ $= \frac{e^{-3} + 4e^3}{-1 - 4}$ $= \frac{e^{-3} + 4e^3}{-5}$ $= -\frac{e^{-3} + 4e^3}{5}$

A5 Ausführliche Lösungen	
<p>e)</p> $y = e^{x^2 - k^2} + 3e^{5k - (k-x)}$ <p>für $x = -k$ gilt:</p> $y = e^{(-k)^2 - k^2} + 3e^{5k - (k - (-k))}$ $= e^{k^2 - k^2} + 3e^{5k - (k+k)}$ $= \underbrace{e^0}_1 + 3e^{5k-2k}$ $= \underline{\underline{1 + 3e^{3k}}}$	<p>f)</p> $y = \frac{x^3 - kx + 1}{x^3}$ <p>für $x = \frac{3}{2k}$ gilt:</p> $y = \frac{\left(\frac{3}{2k}\right)^3 - k\left(\frac{3}{2k}\right) + 1}{\left(\frac{3}{2k}\right)^3} = \frac{\frac{27}{8k^3} - \frac{3}{2} + 1}{\frac{27}{8k^3}} = \frac{\frac{27}{8k^3} - \frac{1}{2}}{\frac{27}{8k^3}}$ $= \frac{\frac{27}{8k^3} - \frac{1}{2}}{\frac{27}{8k^3}} = 1 - \frac{8k^3}{54} = 1 - \frac{4k^3}{27}$

A6 Aufgabe	
Bestimmen Sie den Klammerausdruck	
a) $a^n + a^{4-n} + a^{2n} = a^{2n} (\dots)$	b) $a^{k-2} + a^{3k} + a^{-k-1} = a^{-k} (\dots)$
c) $a^3 + a^{1-n} + a^{n+4} = a^{n+3} (\dots)$	d) $\frac{3}{2}x^4 + \frac{3}{4}x^3 + \frac{1}{8}x^2 = \frac{1}{8}x^2 (\dots)$
e) $e^{3x} - 2e^{-x} = e^{-x} (\dots)$	f) $ke^{2x} - 2e^{x+1} = e^x (\dots)$

A6 Ausführliche Lösungen	
a)	$a^n + a^{4-n} + a^{2n}$ $= a^{2n} \left(\frac{a^n}{a^{2n}} + \frac{a^{4-n}}{a^{2n}} + \frac{a^{2n}}{a^{2n}} \right)$ $= a^{2n} (a^{-n} + a^{4-3n} + 1)$
b)	$a^{k-2} + a^{3k} + a^{-k-1}$ $= a^{-k} \left(\frac{a^{k-2}}{a^{-k}} + \frac{a^{3k}}{a^{-k}} + \frac{a^{-k-1}}{a^{-k}} \right)$ $= a^{-k} (a^{2k-2} + a^{4k} + a^{-1})$

A6 Ausführliche Lösungen	
c)	$a^3 + a^{1-n} + a^{n+4}$ $= a^{n+3} \left(\frac{a^3}{a^{n+3}} + \frac{a^{1-n}}{a^{n+3}} + \frac{a^{n+4}}{a^{n+3}} \right)$ $= a^{n+3} (a^{-n} + a^{-2n-2} + a)$
d)	$\frac{3}{2}x^4 + \frac{3}{4}x^3 + \frac{1}{8}x^2$ $= \frac{1}{8}x^2 \left(\frac{\frac{3}{2}x^4}{\frac{1}{8}x^2} + \frac{\frac{3}{4}x^3}{\frac{1}{8}x^2} + \frac{\frac{1}{8}x^2}{\frac{1}{8}x^2} \right)$ $= \frac{1}{8}x^2 (12x^2 + 6x + 1)$

A6 Ausführliche Lösungen	
e)	$e^{3x} - 2e^{-x} = e^{-x} \left(\frac{e^{3x}}{e^{-x}} - \frac{2e^{-x}}{e^{-x}} \right)$ $= e^{-x} (e^{4x} - 2)$
f)	$ke^{2x} - 2e^{x+1} = e^x \left(\frac{ke^{2x}}{e^x} - \frac{2e^{x+1}}{e^x} \right)$ $= e^x (ke^x - 2e)$