

Lösungen Potenzen III

Ergebnisse:

E1 Ergebnisse					
a)	$\frac{a^6}{a^3} = a^3$	b)	$\frac{(k-3)^4}{(3-k)^3} = 3-k$	c)	$\frac{x^{2n+1}}{x^n} = x^{n+1}$
d)	$\frac{15e^{x+1}}{5e^x} = 3e$	e)	$\frac{x^4}{x^7} = x^{-3}$	f)	$\frac{2a^{1-2n}}{4a^{n+1}} = \frac{1}{2}a^{-3n}$

E2 Ergebnisse					
a)	$\frac{a^4b^{n+3}}{a^nb^{2n-1}} = (ab)^{4-n}$	b)	$\frac{4^{x+2}}{16} = 4^x$	c)	$\frac{81}{3^{x+3}} = 3^{1-x}$
d)	$\frac{(a-b)^3}{(a-b)^{n-1}} = (a-b)^{4-n}$	e)	$\frac{(ab)^3 \cdot (xy)^2}{x^2y \cdot a^4b^2} = a^{-1}by$	f)	$\frac{a^{n+1}}{a^n} = a$

E3 Ergebnisse					
a)	$\frac{10^3}{2^3} = 5^3$	b)	$\frac{2,5^4}{0,5^4} = 5^4$	c)	$\frac{(10ab)^k}{(4b)^k} = \left(\frac{5}{2}a\right)^k$
d)	$\frac{5^3}{(-0,2)^3} = -25^3$	e)	$\frac{(4-x^2)^n}{(2-x)^n} = (2+x)^n$	f)	$\frac{(a^2-b^2)^3}{(a-b)^3} = (a+b)^3$

E4 Ergebnisse					
a)	$\frac{c^6}{(-c)^6} + 1 = 2$	b)	$\frac{(c-1)^{n-1}}{(c^2-1)^{n-1}} = \frac{1}{(c+1)^{n-1}}$	c)	$\left(\frac{a}{b}\right)^n \cdot \frac{a}{b} = \left(\frac{a}{b}\right)^{n+1}$
d)	$\left(\frac{-1}{a-b}\right)^3 = -(a-b)^{-3}$	e)	$\left(\frac{x}{2}\right)^3 : \left(\frac{x}{3}\right) = \frac{3}{8}x^2$	f)	$\frac{(a^{2n}-b^{2n})^2}{(a^n-b^n)^2} = (a^n+b^n)^2$

E5 Ergebnisse					
a)	$(-5^2)^3 = -5^6$	b)	$3(c^4)^3 - 6c^{12} = -3c^{12}$	c)	$(3b^2c^{n-1})^4 = 81b^8c^{4n-4}$
d)	$\left(\frac{7a^2}{49b^3}\right)^2 = \frac{a^4}{49b^6}$	e)	$\left(\frac{-1}{c^3}\right)^{2n} = \frac{1}{c^{6n}}$	f)	$(3b^{n+1} \cdot c^{n-1})^2 = 9b^{2n+2}c^{2n-2}$

E6	Ergebnisse
a)	$(x^2y^3z^2)^5 = x^{10}y^{15}z^{10}$
b)	$(0,5e^{x+2})^2 = 0,25e^{2x+4}$
c)	$(a^3 - ab^2)(a+b)^2 = (a-b) \cdot a \cdot (a+b)^3$
d)	$\frac{(2x-4)^5}{(2-x)^3} = -32(x-2)^2$
e)	$\frac{[(x-y)^2]^k}{(x^2-y^2)^k} = \left(\frac{x-y}{x+y}\right)^k$
f)	$\frac{(4ab)^4 \cdot 5}{(6a^2)^4 \cdot b^4} = \frac{80}{81a^4}$

Potenzgesetze

$a^m \cdot a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$	$a^n \cdot b^n = (a \cdot b)^n$	$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$
$(a^n)^m = a^{n \cdot m}$	$\sqrt[n]{a^m} = a^{\frac{m}{n}}$	$a^0 = 1$	$\frac{1}{a^n} = a^{-n}$

Ausführliche Lösungen :

A1	Aufgabe					
	Vereinfachen Sie					
	a)	$\frac{a^6}{a^3}$	b)	$\frac{(k-3)^4}{(3-k)^3}$	c)	$\frac{x^{2n+1}}{x^n}$
d)	$\frac{15e^{x+1}}{5e^x}$	e)	$\frac{x^4}{x^7}$	f)	$\frac{2a^{1-2n}}{4a^{n+1}}$	

A1	Ausführliche Lösungen					
	a)	$\frac{a^6}{a^3} = a^{6-3}$ $= \underline{\underline{a^3}}$	b)	$\frac{(k-3)^4}{(3-k)^3} = \frac{[(-1)(3-k)]^4}{(3-k)^3}$ $= \frac{[(-1)^4(3-k)^4]}{(3-k)^3}$ $= \frac{(3-k)^4}{(3-k)^3}$ $= (3-k)^{4-3}$ $= \underline{\underline{3-k}}$	c)	$\frac{x^{2n+1}}{x^n} = x^{2n+1-n}$ $= \underline{\underline{x^{n+1}}}$
	d)	$\frac{15e^{x+1}}{5e^x} = \frac{15}{5} \cdot \frac{e^{x+1}}{e^x}$ $= 3 \cdot e^{x+1-x}$ $= \underline{\underline{3e}}$	e)	$\frac{x^4}{x^7} = x^{4-7}$ $= x^{-3}$ $= \underline{\underline{\frac{1}{x^3}}}$	f)	$\frac{2a^{1-2n}}{4a^{n+1}} = \frac{1}{2} \cdot a^{1-2n-(n+1)}$ $= \frac{1}{2} a^{1-2n-n-1}$ $= \frac{1}{2} a^{-3n}$ $= \underline{\underline{\frac{1}{2a^{3n}}}}$

A2	Aufgabe					
	Vereinfachen Sie					
	a)	$\frac{a^4 b^{n+3}}{a^n b^{2n-1}}$	b)	$\frac{4^{x+2}}{16}$	c)	$\frac{81}{3^{x+3}}$
d)	$\frac{(a-b)^3}{(a-b)^{n-1}}$	e)	$\frac{(ab)^3}{x^2 y} \cdot \frac{(xy)^2}{a^4 b^2}$	f)	$\frac{a^{n+1}}{a^n}$	

A2	Ausführliche Lösungen						
	a)	$\frac{a^4 b^{n+3}}{a^n b^{2n-1}} = a^4 \cdot b^{n+3} \cdot a^{-n} \cdot b^{-(2n-1)}$ $= a^{4-n} \cdot b^{n+3-2n+1}$ $= a^{4-n} \cdot b^{4-n}$ $= \underline{\underline{(ab)^{4-n}}}$			b)	$\frac{4^{x+2}}{16} = \frac{4^{x+2}}{4^2}$ $= 4^{x+2-2}$ $= \underline{\underline{4^x}}$	
	c)	$\frac{81}{3^{x+3}} = \frac{3^4}{3^{x+3}}$ $= 3^{4-(x+3)}$ $= 3^{4-x-3}$ $= \underline{\underline{3^{1-x}}}$			d)	$\frac{(a-b)^3}{(a-b)^{n-1}} = (a-b)^3 \cdot (a-b)^{-(n-1)}$ $= (a-b)^3 \cdot (a-b)^{-n+1}$ $= (a-b)^{3-n+1}$ $= \underline{\underline{(a-b)^{4-n}}}$	
e)	$\frac{(ab)^3}{x^2 y} \cdot \frac{(xy)^2}{a^4 b^2}$ $= \frac{a^3 \cdot b^3 \cdot x^2 \cdot y^2}{x^2 \cdot y \cdot a^4 \cdot b^2}$ $= a^3 \cdot b^3 \cdot x^2 \cdot y^2 \cdot x^{-2} \cdot y^{-1} \cdot a^{-4} \cdot b^{-2}$ $= a^{3-4} \cdot b^{3-2} \cdot x^{2-2} \cdot y^{2-1}$ $= a^{-1} \cdot b^1 \cdot x^0 \cdot y^1$ $= \underline{\underline{a^{-1}by}}$			f)	$\frac{a^{n+1}}{a^n} = a^{n+1-n}$ $= a^1$ $= \underline{\underline{a}}$		

A3 Aufgabe					
Vereinfachen Sie					
a)	$\frac{10^3}{2^3}$	b)	$\frac{2,5^4}{0,5^4}$	c)	$\frac{(10ab)^k}{(4b)^k}$
d)	$\frac{5^3}{(-0,2)^3}$	e)	$\frac{(4-x^2)^n}{(2-x)^n}$	f)	$\frac{(a^2-b^2)^3}{(a-b)^3}$

A3 Ausführliche Lösungen					
a)	$\frac{10^3}{2^3} = \left(\frac{10}{2}\right)^3$ $= 5^3$ $= \underline{\underline{125}}$	b)	$\frac{2,5^4}{0,5^4} = \left(\frac{2,5}{0,5}\right)^4$ $= \underline{\underline{5^4}}$		
c)	$\frac{(10ab)^k}{(4b)^k} = \left(\frac{10ab}{4b}\right)^k$ $= \underline{\underline{\left(\frac{5}{2}a\right)^k}}$	d)	$\frac{5^3}{(-0,2)^3} = \left(\frac{5}{-0,2}\right)^3$ $= (-25)^3$ $= \underline{\underline{-25^3}}$		
e)	$\frac{(4-x^2)^n}{(2-x)^n} = \left(\frac{4-x^2}{2-x}\right)^n$ $= \left[\frac{\cancel{(2-x)}(2+x)}{\cancel{(2-x)}}\right]^n$ $= \underline{\underline{(2+x)^n}}$	f)	$\frac{(a^2-b^2)^3}{(a-b)^3} = \left(\frac{a^2-b^2}{a-b}\right)^3$ $= \left[\frac{\cancel{(a-b)}(a+b)}{\cancel{(a-b)}}\right]^3$ $= \underline{\underline{(a+b)^3}}$		

A4	Aufgabe					
	Vereinfachen Sie					
	a)	$\frac{c^6}{(-c)^6} + 1$	b)	$\frac{(c-1)^{n-1}}{(c^2-1)^{n-1}}$	c)	$\left(\frac{a}{b}\right)^n \cdot \frac{a}{b}$
d)	$\left(\frac{-1}{a-b}\right)^3$	e)	$\left(\frac{x}{2}\right)^3 : \left(\frac{x}{3}\right)$	f)	$\frac{(a^{2n}-b^{2n})^2}{(a^n-b^n)^2}$	

A4	Ausführliche Lösungen							
	a)	$\frac{c^6}{(-c)^6} + 1 = \left(\frac{c}{-c}\right)^6 + 1$ $= (-1)^6 + 1$ $= 1 + 1$ $= \underline{\underline{2}}$	b)	$\frac{(c-1)^{n-1}}{(c^2-1)^{n-1}} = \left(\frac{c-1}{c^2-1}\right)^{n-1}$ $= \left[\frac{1 \cdot \cancel{(c-1)}}{\cancel{(c-1)}(c+1)}\right]^{n-1}$ $= \left(\frac{1}{c+1}\right)^{n-1} = \underline{\underline{\frac{1}{(c+1)^{n-1}}}}$	c)	$\left(\frac{a}{b}\right)^n \cdot \frac{a}{b} = \left(\frac{a}{b}\right)^n \cdot \left(\frac{a}{b}\right)^1$ $= \underline{\underline{\left(\frac{a}{b}\right)^{n+1}}}$	d)	$\left(\frac{-1}{a-b}\right)^3 = \frac{(-1)^3}{(a-b)^3}$ $= \frac{-1}{(a-b)^3}$ $= \underline{\underline{-(a-b)^{-3}}}$
	e)	$\left(\frac{x}{2}\right)^3 : \left(\frac{x}{3}\right) = \frac{x^3}{2^3} : \frac{x}{3}$ $= \frac{3x^3}{8x}$ $= \underline{\underline{\frac{3}{8}x^2}}$	f)	$\frac{(a^{2n}-b^{2n})^2}{(a^n-b^n)^2} = \left(\frac{a^{2n}-b^{2n}}{a^n-b^n}\right)^2$ $= \left[\frac{\cancel{(a^n-b^n)}(a^n+b^n)}{\cancel{(a^n-b^n)}}\right]^2$ $= \underline{\underline{(a^n+b^n)^2}}$				

A5 Aufgabe			
Vereinfachen Sie			
a)	$(-5^2)^3$	b)	$3(c^4)^3 - 6c^{12}$
c)	$(3b^2c^{n-1})^4$	d)	$\left(\frac{7a^2}{49b^3}\right)^2$
e)	$\left(\frac{-1}{c^3}\right)^{2n}$	f)	$(3b^{n+1} \cdot c^{n-1})^2$

A5 Ausführliche Lösungen			
a)	$\begin{aligned} (-5^2)^3 &= [(-1) \cdot 5^2]^3 \\ &= [(-1)^3 \cdot 5^{2 \cdot 3}] \\ &= -1 \cdot 5^6 \\ &= \underline{\underline{-5^6}} \end{aligned}$	b)	$\begin{aligned} 3(c^4)^3 - 6c^{12} \\ &= 3 \cdot c^{12} - 6c^{12} \\ &= \underline{\underline{-3c^{12}}} \end{aligned}$
c)	$\begin{aligned} (3b^2c^{n-1})^4 &= 3^4 \cdot b^{2 \cdot 4} \cdot c^{(n-1) \cdot 4} \\ &= 3^4 \cdot b^8 \cdot c^{4n-4} \\ &= \underline{\underline{81b^8c^{4n-4}}} \end{aligned}$	d)	$\begin{aligned} \left(\frac{7a^2}{49b^3}\right)^2 &= \left(\frac{a^2}{7b^3}\right)^2 \\ &= \frac{a^4}{7^2 \cdot b^6} \\ &= \underline{\underline{\frac{a^4}{49b^6}}} \end{aligned}$
e)	$\begin{aligned} \left(\frac{-1}{c^3}\right)^{2n} &= \frac{(-1)^{2n}}{c^{3 \cdot 2n}} \\ &= \frac{1}{c^{6n}} = \underline{\underline{c^{-6n}}} \end{aligned}$	f)	$\begin{aligned} (3b^{n+1} \cdot c^{n-1})^2 &= 3^2 \cdot b^{2(n+1)} \cdot c^{2(n-1)} \\ &= \underline{\underline{9b^{2n+2}c^{2n-2}}} \end{aligned}$

A6	Aufgabe		
	Vereinfachen Sie		
	a)	b)	c)
d)	e)	f)	

$$\begin{array}{|c|c|c|} \hline \text{a)} & (x^2 y^3 z^2)^5 & \text{b)} & (0,5e^{x+2})^2 & \text{c)} & (a^3 - ab^2)(a+b)^2 \\ \hline \text{d)} & \frac{(2x-4)^5}{(2-x)^3} & \text{e)} & \frac{[(x-y)^2]^k}{(x^2-y^2)^k} & \text{f)} & \frac{(4ab)^4}{(6a^2)^4} \cdot \frac{5}{b^4} \\ \hline \end{array}$$

A6	Ausführliche Lösungen		
	a)	b)	
	c)	d)	
e)	f)		

$$\begin{array}{|c|c|c|} \hline \text{a)} & (x^2 y^3 z^2)^5 = x^{2 \cdot 5} \cdot y^{3 \cdot 5} \cdot z^{2 \cdot 5} \\ & = \underline{\underline{x^{10} y^{15} z^{10}}} \\ \hline \text{b)} & (0,5e^{x+2})^2 = 0,5^2 \cdot e^{2(x+2)} \\ & = \underline{\underline{0,25e^{2x+4}}} \\ \hline \text{c)} & (a^3 - ab^2)(a+b)^2 \\ & = a(a^2 - b^2)(a+b)^2 \\ & = a(a-b)(a+b)(a+b)^2 \\ & = \underline{\underline{a(a-b)(a+b)^3}} \\ \hline \text{d)} & \frac{(2x-4)^5}{(2-x)^3} = \frac{[2(x-2)]^5}{(2-x)^3} \\ & = \frac{[2(x-2)]^5}{[(-1)(x-2)]^3} \\ & = \frac{2^5(x-2)^5}{-1(x-2)^3} \\ & = \underline{\underline{-32(x-2)^2}} \\ \hline \text{e)} & \frac{[(x-y)^2]^k}{(x^2-y^2)^k} = \frac{[(x-y)^2]^k}{[(x-y)(x+y)]^k} \\ & = \frac{[(x-y)(x-y)]^k}{[(x-y)(x+y)]^k} \\ & = \underline{\underline{\left(\frac{x-y}{x+y}\right)^k}} \\ \hline \text{f)} & \frac{(4ab)^4}{(6a^2)^4} \cdot \frac{5}{b^4} = \frac{4^4 \cdot a^4 \cdot \cancel{b^4} \cdot 5}{6^4 \cdot a^8 \cdot \cancel{b^4}} \\ & = \frac{(2 \cdot 2)^4 \cdot 5}{(2 \cdot 3)^4 \cdot a^4} \\ & = \left(\frac{\cancel{2} \cdot 2}{\cancel{2} \cdot 3}\right)^4 \cdot \frac{5}{a^4} \\ & = \frac{2^4 \cdot 5}{3^4 \cdot a^4} \\ & = \underline{\underline{\frac{80}{81a^4}}} \\ \hline \end{array}$$